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Instructional Scaffolds in Mathematics Instruction for English Learners with Learning Disabilities: An Exploratory Case Study

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As today's classrooms become more and more diverse, there is a growing need to explore the intersection between English Learners (ELs) and students with learning disabilities (LD) in the content-specific instruction of mathematics problem solving. The aim of this study was to determine which types of instructional scaffolds may be used by math teachers to effectively support ELs with LD learning multiplicative reasoning. To this end, we employed an exploratory case study based on a frequency count analysis of four scaffold types used by the students and the teacher in their sessions. The results showed that kinesthetic and linguistic scaffolds were the most beneficial for helping ELs with LD to cultivate mathematical thinking with both concrete and abstract units, while also helping to increase the sophistication of their mathematical content-language usage. In combination with small-group interactions, these scaffolds provide an effective instructional method for improving multiplicative reasoning among ELs with LD.

Keywords: **Number Concepts and Operations, Mathematical Knowledge for Teaching, Elementary, English Learners, Scaffolds**

Introduction

English learners (ELs) are the fastest-growing student subgroup in the United States (Morita-Mullaney & Singh, 2019; National Center for Education Statistics, 2017). Despite federal and state requirements to meet their unique linguistic needs, ELs are often situated in schools that are under-resourced and have few EL personnel and/or programs. As a result, EL students may end up being classified as and receiving services as students with learning disabilities (LD) as a substitution for English language development services. Often, the instructional services provided for EL students within a special education program are not specifically geared to their English language learning needs (Collier, 2011; Kangas, 2019). Furthermore, established EL programs and general education classrooms often neglect the content area of mathematics, with most attention being applied toward English language development (de Araujo, Roberts, Willey, & Zahner, 2018), positioning math as a universally accessible or language-free content (Lee, Quinn, & Valdés, 2013; Torres-Velasquez & Rodriguez, 2005).

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According to the section on English language acquisition in Title III of the Elementary and Secondary Education Act (ESEA) reauthorized as the Every Student Succeeds Act (ESSA) in 2015, U.S. schools are accountable for the improvement of all children, including those with "disability, recently arrived ELs, and long-term ELs" (Non-Regulatory Guidance, 2016, p. 4). As such, students with limited English proficiency, or English Learners, as referred to in ESSA, must meet benchmark achievement goals (pass/do not pass) and make adequate annual growth in English language proficiency and mathematics, not just English language arts.

This requirement poses a dilemma, given the emphasis on language arts mentioned above. Thus, in order to meet federal and state accountabilities regulations set forth for dually classified ELs (EL and special education), schools need to provide linguistically appropriate and content-specific school-level intervention support in a timely manner to promote the academic performance of dually classified ELs and address persistent achievement gaps in math (August, Spencer, Fenner, & Kozik, 2012; de Araujo et al., 2018; Lee et al., 2013). Therefore, understanding how to provide appropriate interventions for ELs is increasing in importance.

Teaching Math Content and Related Language

ELs experience a dually challenging task of learning the language along with academic content (Gerena & Keiler, 2012; Kangas, 2019; Short & Fitzsimmons, 2007). Although they may appear to be verbally fluent in English, they may still struggle with complex academic material that requires producing specific academic discourse or vocabulary (Gerena & Keiler, 2012; Morita-Mullaney & Stallings, 2018; Olsen, 2010) that differs from social language in use. In her Academic Literacy in Mathematics framework, Moschkovich (2015) argues that there are literacy and language components to learning math and that their inclusion is necessary for lesson preparation and instruction. Specifically, using Gee's (2015) work on discourse, Moschkovich (2015) points out that math has a particular syntax, structure, and vocabulary that math educators need to understand and employ with and among ELs.

Scholars in the fields of English learning and bilingual education have recommended the use of instructional scaffolds to help convey meaning to students at varying levels of English proficiency. Scaffolds may be visual/graphic, linguistic, interactive, and kinesthetic (Gibbons, 2014; Gottlieb, 2016), and are used by students and teachers before, during, and after instruction to support content and content-specific language learning. Thus, scaffolds are important considerations in the planning of math instruction for dually classified ELs or ELs with learning disabilities (McGhee, 2011).

In this study, we examined four kinds of instructional scaffolding to analyze the mathematics instructional discourse exchanges between a teacher and ELs with LD within the context of a small-group constructivist-oriented learning environment learning environment. We posed the following research questions:

1. What types of scaffolds do teachers and dually classified ELs make in multiplicative reasoning during instruction and assessment activities?
2. How do teachers regulate language usage and scaffolding to facilitate the multiplicative reasoning of dually classified ELs?

Statement of the Problem

In addition to ELs being the fastest growing student subgroup in the United States, accounting for 4.6 million students (National Center for Education Statistics, 2017), a subset within this population are also identified as students in special education. ELs with disabilities represent 13.8% of that 4.6 million, constituting around 635,000 with this dual classification. This poses a unique challenge because although the fields of EL, or bilingual education, and special education have definitions of EL and learning disabilities, respectively, how these two identifiers intertwine in the context of providing instruction in the classroom has received scant attention.

The EL and special education fields have addressed the possible overrepresentation of ELs as special education due to ignoring the effects of second language acquisition on ELs' academic progress (Association for Supervision and Development & Centers for Disease Control and Prevention, 2014; Kangas, 2017) or the absence of available EL instructional services driving referrals (Kangas, 2014). Alternatively, underrepresentation of ELs in special education has also been studied, where initial identification is avoided in favor of permitting time for English mastery to take hold (Sullivan, 2011). Yet, little work to date has identified what types of instruction are furnished to dually classified ELs with LD.

Literature Review

Students who are dually classified as ELs in special education fall at the crossroads of English language learning and a specific learning disability, making instructional service provisions challenging and often unequal, with special education provisions often taking precedence with limited consideration of students' language proficiency in English and other home or heritage languages (Collier, 2011; Kangas, 2014, 2019).

As a result, the individual EL student's distinct English proficiency level and specific special education identification do not smoothly guide what instructional practices are best suited for learning content such as math and its related language or discourse. To investigate the intersections of these issues, the following literature review is divided into two sections. First, literature regarding dual classification of special education and EL is examined. Second, research pertaining to the complexity of math instruction for dually classified students is reviewed.

Dual Classification of Special Education and EL

Operationally defining an EL in special education is complicated by the moderating variables of a students' age, language background, levels of language proficiency, and socioeconomic status (Linquanti & Cook, 2015). Watkins and Liu (2013) made an attempt to define a dually classified EL in simple terms, stating that an ELL[1] with disabilities is a student who is eligible for both special education and English as a second language (ESL) or bilingual education services. There are different identification issues associated with each service, creating variability in the definition of an EL with disabilities across the country.

1 The authors use the term *ELL* or *English language learner*, which is also used in the literature to describe students with a language other than English in their background. ESSA, however, uses the term English Learner (EL).

This definition clearly demonstrates that researchers in EL/bilingual education and special education have mostly focused on identification practices of dually classified students, and not on instructional provisions (Kangas, 2019). This preoccupation with which EL is *in* and/or *out* of special education has led to studies on disproportionality of under- and over-identification of ELs with learning disabilities (Artiles, Rueda, Salazar, & Higareda, 2005; Brown & Ault, 2015; Sullivan, 2011).

Math Instruction for Dually Classified Students

Instructional provisions for dually classified ELs with LD is an emerging field. Few evidence-based and high-impact strategies have been identified for students with this particular type of dual classification (Klingner, Artiles, & Barletta, 2006; Lesaux, Kieffer, Faller, & Kelley, 2010; More, Spies, Morgan, & Baker, 2016; Sáenz, Fuchs, & Fuchs, 2005; Shyyan, Thurlow, & Liu, 2006), and only one of them has specifically addressed the content area of math (Shyyan et al., 2006). Shyyan et al. (2006) found that ELs placed greater priority on evidence-based math strategies, whereas their teachers placed a greater emphasis on other content areas, demonstrating a mismatch between what dually classified ELs with LD articulate is of most value.

Conceptual Framework

According to a constructivist framework for learning, students and instructors mediate understanding to move to new and incremental understanding around target content (Vygotsky, 1962). The instructor plays a key role in facilitating this framework as he or she attends to what students understand (Tzur et al., 2013; Xin et al., 2017). According to Tzur et al. (2013):

> In our constructivist framework, to solve a task, a child has to (a) assimilate it into the situation part of an existing scheme, (b) identify the quantities (mental objects) involved, (c) set a goal compatible with the question, (d) initiate mental activities on those quantities that (in the child's mind) correspond to the depicted relationships, and (e) constantly compare the actual effects of the activity to the goal to determine the conclusion of the activity (p. 87).

These five areas work together to build the student's competency in target math content by working methodically through these conceptual steps based on how students respond to new content.

While we chose a constructivist framework for the present study, the most commonly used approach to address the needs of ELs is the use of accommodations for tests, which include changing the test itself, the test response format, or the test process (Abedi, Hofstetter, & Lord, 2004). According to Chiu and Pearson (1999), special education and limited English proficient (LEP) students' (or ELs') standardized achievement test scores can increase when they get appropriate accommodations, yet we know little about how accommodations support daily instructional activities.

The most common accommodations used during instruction include providing extra time, using a bilingual dictionary, and facilitation within a small group or specialized attention from an EL specialist. Yet, there are limited instructional shifts on the part of the teacher because the accommodations are applied after instruction instead of before and during instruction. A promising paradigmatic shift in the field of English learning is the use of supports or scaffolds – the focus on this study – whereby teachers create the conditions for comprehensible input of content and language by using scaffolds *during* instruction (Gibbons, 2014; Gottlieb, 2016; Krashen, 1998).

Scaffolding

In the teaching-learning framework, scaffolding is a central notion adapted from Gibbons (2002, 2014) and supported by a constructivist theory of learning. Scaffolding is an essential support to "enable children [ELs] to perform tasks independently that previously they could perform only with the assistance or guidance of the teacher" (Gibbons, 2002, p. vii). Gibbons (2002, 2014) suggested that scaffolding can also be used for English language teaching to ELs within general education classrooms, where they spend the majority of their school day. While the use of scaffolds has been widely studied within special education (Stone, 1998; Wood, Bruner, & Ross, 1976), this is a relatively new approach within English learning (Gibbons, 2002, 2014).

Scaffolds are strategies that support the delivery of target content with explicit inclusion of a given scaffold appropriate for each ELs' level of English proficiency and, in this case, the added dimension of a learning disability. Gottlieb (2016) described four types of instructional scaffolds that teachers can use, and students can engage in, to foster understanding of target content and related language: visual, linguistic, interactive, and kinesthetic (Gottlieb, 2013).

Visual scaffolding. Visual scaffolding involves the use of drawings or photographs to connect English words, phrases, and sentences to visual images, and assists ELs in learning the target content (Lei, Xin, Morita-Mullaney, & Tzur, 2018). This approach makes complex ideas feel more accessible to students and makes the language more memorable while providing comprehensible input of the target content (McCloskey, 2005). A variety of visual supports can be used to build students' visual experience in the classroom, including manipulatives, real-life objects, and multimedia material (Carrasquillo & Rodrigues, 2002; Gottlieb, 2012).

Linguistic scaffolding. Linguistic scaffolding provides effective and responsive support for students' language output performance, which requires teachers to use language that is comprehensible to students when providing them with new and more sophisticated knowledge, including the use of a slower rate of speech, simplified vocabulary, or cycling speech with consistent reinforcement of a target set of words (Bradley & Reinking, 2011; Gibbons, 2003).

Interactive scaffolding. Interactive scaffolding involves a strategic back-and-forth between teachers and students or among students to facilitate comprehension of content and related language use. Goffman (1967) proposed the idea of "interactionism," which relates "only to those aspects of 'context' that are directly observable and to such immediate links between individuals as their 'roles,' 'obligations,' 'face-to-face encounters'" (pp. 31–49). An example of instructional support for interaction involves both students and teachers taking on active roles in pair work and small-group work (Gibbons, 2008).

Kinesthetic scaffolding. Asher (1969) first introduced a strategy called Total Physical Response, which directly relates to kinesthetic scaffolding. This approach requires students to listen to a language command that may or may not be stated in their heritage language, and follow it using a physical action immediately with no expectation of speech production (Asher, 1969). This process lowers their anxiety, allowing them to produce content knowledge nonverbally, but with a related object or physical movement. Brand, Favazza, and Dalton (2012) suggested that students who use kinesthetic scaffolding benefit from "sign language, translation into another language, gestures" during sessions (p. 139), while not being restricted from participating in classroom activities due to their lower levels of English proficiency.

In summary, visual scaffolding is the most frequently used scaffold with ELs as it is readily available and simple to employ (Walqui, 2010; Walqui & vanLier, 2010). Linguistic and interactive scaffolds, on the other hand, are not seen as helpful as ELs "lack" supposed English proficiency. Finally, kinesthetic scaffolds, which involve movement, are often disregarded as movement and motion may not be deemed as developmentally appropriate for older students.

Regardless of the instructional scaffold(s) used, discourse or language is a part of the delivery of content. Therefore, how teachers use their language is critical for dually classified ELs. For instance, Bishop and Whitacre (2010) coded the teacher's and the student's discourse moves as "give moves" for providing information and "demand moves" for requesting information. Xin, Liu, Jones, Tzur, and Si (2016) used a similar structure to code the teacher's and the student's discourse moves, using "low," "potentially high," and "high" to distinguish between three levels of intellectual work.

In consideration of instructional scaffolds and related discourse moves, the conceptual framework that guides our study is shown in Figure 1. As illustrated, instructional scaffolds and math content occur in tandem, undergirded by thoughtful preparation of content and related scaffolds.

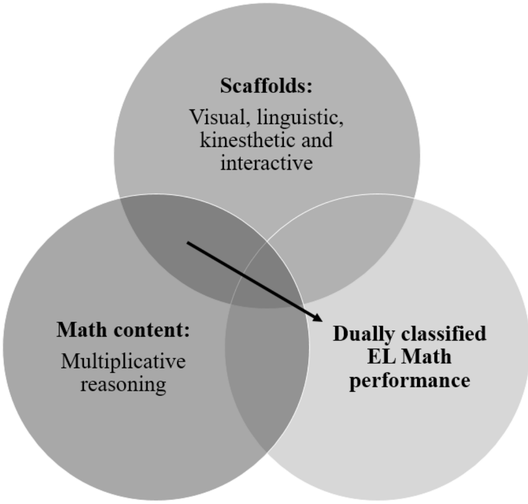

***Figure 1.* Instructional scaffolds in math instruction for dually classified ELs.**

Research Methodology

This study was conducted within the larger context of the National Science Foundation-funded *Nurturing Multiplicative Reasoning in Students With Learning Disabilities/Difficulties* project[2] (Xin, Tzur, & Si, 2008). With a constructivist view of learning that is consistent with reform-based instruction, a teaching experiment (Steffe & Thompson, 2000) was designed and carried out to promote the multiplicative reasoning of seven pairs of fourth and fifth graders with LD. The present study focuses on one student, Eliza, a dually classified EL with LD, during an instructional intervention with her teacher using a constructivist approach.

Mode of Inquiry

An exploratory case study was used to examine the scaffolds used by teachers and appropriated by dually classified ELs. Yin (2014) defined a case study as "an empirical inquiry that investigates a contemporary phenomenon (the 'case') within its real-life context, especially when the boundaries between phenomenon and context may not be clearly evident" (p. 16). The present study investigated the interplay between teacher and student in mathematics instruction from a constructivist perspective of learning (Vygotsky, 1962). The teacher (one of the authors) worked closely with Eliza (the dually classified EL) and another native English speaker with LD in seven sessions for around 40 minutes each over the course of six months.

As stated earlier, constructivism is a philosophy of learning that focuses on individuals actively participating in learning rather than passively receiving knowledge (Gunning, 2010). From this perspective, the learning process can only occur when the learners are actively engaged in integrating new knowledge with existing knowledge (Tracey & Morrow, 2012). As such, a constructivist teaching frame identifies the schema and backgrounds of students related to the academic content and then initiates and mediates related activities so students can build their understanding incrementally. The instructor plays a key role in facilitating this meaning-making at key mental intervals to ensure greater connection to target content (Tzur et al., 2013).

Setting and Context of the Study

The study took place at an elementary school in the midwestern United States in the Matthias District (pseudonym), an urban school district. The school, Monon Elementary (pseudonym), has a total enrollment of 398 students and is one of the most populated elementary buildings in a district of 9 elementary schools. Thirty-four state-licensed teachers serving at Monon elementary (see Table 1). Thirty-two percent of the students are children of color, with a distribution of 64% White, 16% Hispanic, 9% Black, 6% Multiracial, 1% Asian, 1% Native American, and 3% as non-identified. Fifty-one percent of the students at the school are eligible for free and reduced-cost lunch, indicating a high density of poverty and additional Title I resources to address English language arts reading needs of its students, but not math. The proportion of ELs for this school district is moderately high at 12.6% of

2 This research was supported by the National Science Foundation under Grant DRL 0822296. The opinions expressed do not necessarily reflect the views of the foundation.

the total school population, a larger representation than the other elementary schools in the district (Indiana Department of Education, 2006). For special education, 15.5% of the students are formally identified, which suggests possible over-identification.

Table 1. ***Thirty-Four State-Licensed Teachers Serving at Monon Elementary***

Year	Years of teaching	Number of teachers
2008-09	0-5 years	4
2008-09	6-10 years	3
2008-09	11-15 years	2
2008-09	16-20 years	4
2008-09	20+ years	21

Each teaching experiment lesson was designed based on an assessment of the student's level of understanding of the given math content from the previous session. In each session, the instructor provided a constructivist framework to promote the EL's progress toward multiplicative reasoning (Tzur, Xin, Kenney, & Guebert, 2010) and problem-solving (Xin, 2012).

Participants

The study includes two types of participants: (a) the instructor conducting the math intervention and (b) the target students of the math intervention.

Instructor. The instructor for the intervention has over 20 years of experience in teaching elementary, middle, and high school mathematics (including remedial math). He is bilingual in English and Hebrew; he came to the United States as an adult and was an English learner himself with his dominant language being Hebrew. In addition to his K-12 teaching experience, he has served in various faculty positions in the United States, including the development of collaboration with scholars from the field of special education and expertise in teaching and researching mathematics involving students with learning disabilities. He had employed this particular math intervention in three other studies (Tzur et al., 2013; Xin et al., 2016, 2017), as well as in numerous public schools – as the lead teacher, a co-teacher, or a coach working with elementary teachers who implemented the intervention.

Students. Our two student participants attended an after-school program: (a) Eliza, a fifth-grade dually classified EL with LD and (b) Leslie, a fifth-grade native English speaker with LD.

According to Eliza's IEP, she was included in a general education class setting for 50% of the time and received 45 minutes of math instruction in the resource room each day from different math instructors. She was placed in a learning support classroom for reading, English language arts, and math. Eliza's intellectual functioning score was 69 on the Otis-Lennon School Ability Test (with a 95% confidence interval ranged between 68 to 132; TestPrep-Online, n.d.). Eliza had been in the special education program for four years.

The fifth-grade native English student with LD (Leslie) worked as a group partner with Eliza during each session. Leslie's intellectual functioning was 80 on the full-scale Otis-Lennon test, ranked over one standard deviation below the norm. Leslie had been in the special education program for four years as well.

Data Sources

The sources of data were teaching videos and on-site field notes taken during observations of the teaching experiment session. The instruction involved students engaging in solving multiplicative word problems in the context of a turn-taking, a platform game we called "Please Go and Bring for Me …" (PGBM). The basic version of the game involves sending a student to a box with Unifix Cubes in the classroom with a task of producing and bringing back a tower made of a few cubes (fewer than 10 cubes). After taking 2-9 trips to the box of cues and being asked to bring the same-size towers, students are asked how many towers (i.e., composite units, CU) they brought, how many cubes each tower has (i.e., unit rate, UR, Xin, 2012), and how many cubes (1's) there are in all. In addition, the teacher also asked students to pose similar problems to the teacher (or the other student) to evaluate their conceptual understanding at a higher level (e.g., the focal student will ask the teacher: "Please go and bring me 3 towers of 4." "How many cubes will you have in all?"). Towards the end of the teaching experiment, students were introduced to Conceptual Model-based Problem Solving (COMPS, Xin, 2012) using a single model equation (i.e., Unit Rate x # of Units = Product [Xin, 2012]) to solve a range of multiplication and division problems. The teaching videos recorded the teacher and focal students. A graduate student who was enrolled in the School Counseling program took the field notes.

Intervention

The PGBM-COMPS is an evidence-based intervention (Xin et al., 2017) based on the Indiana State Math Standards of 2006 for grade 5; the target grade level for the intervention (Indiana Department of Education, 2006). As shown in Table 2, the math intervention focused on math computation and algebraic reasoning. Although algebraic reasoning was not articulated in the 2006 math standards, it is represented in the current Indiana math standards and was incorporated into the PGBM-COMPS intervention (Indiana Department of Education, 2014).

Data Analysis

We coded the discourse among the instructor and both the dually classified EL with LD and a native English speaker with LD with regard to problem solving and reasoning, as well as how they appropriated language to convey their reasoning. The discourse moves were coded into four types of scaffoldings: visual/graphic scaffolding, linguistic scaffolding, interactive scaffolding, and kinesthetic scaffolding. The purpose of this coding method was to answer the first research question: *What types of scaffolds do teachers and dually classified ELs make in multiplicative reasoning during instruction and assessment activities*?

Coding scheme of discourse moves. Using NVivo 11 (QSR International Pty Ltd., 2017), the verbal and nonverbal mathematical communication, as well as their behavior (e.g., using finger counting, creating the mathematical model on scratch paper), was coded for the teacher and the pair of students (Xin et al., 2016). Any unrelated mathematical verbal or nonverbal communication or behavior was not coded as it was not central to the inquiry.

Table 2. ***Indiana Fifth-Grade Math Standards (2006, 2014)***

2006 Indiana 5th grade math standards	2014 Indiana 5th grade math standards	PGBM-COMPS intervention
5.2.1: Solve problems involving multiplication and division of any whole numbers. Example: 2,867 x 34 = ?. Explain your method. COMPUTATION	**5.C.1:** Multiply multi-digit whole numbers fluently using a standard algorithmic approach. COMPUTATION	2006: *yes* 2014: *yes*
5.2.3: Use models to show an understanding of multiplication and division of fractions. Example: Draw a rectangle 5 squares wide and 3 squares high. Shade 4/5 of the rectangle, starting from the left. Shade 2/3 of the rectangle, starting from the top. Look at the fraction of the squares that you have double-shaded and use that to show how to multiply 4/5 by 2/3. COMPUTATION	**5.C.3:** Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. COMPUTATION	2006: *yes* 2014: *yes*
5.2.6: Use estimation to decide whether answers are reasonable in addition, subtraction, multiplication, and division problems. Example: Your friend says that 2,867 x 34 = 20,069. Without solving, explain why you think the answer is wrong. COMPUTATION		2006: *yes*
	5.AT.1: Solve real-world problems involving multiplication and division of whole numbers (e.g. by using equations to represent the problem). In division problems that involve a remainder, explain how the remainder affects the solution to the problem. ALGEBRAIC	2014: *yes*

A scaffolding coding scheme documented two interrelated areas: (a) what the teacher stated and what scaffold he applied while instructing the target math content; and (b) what the students stated in response to math instruction and what scaffold they appropriated. Table 3 illustrates the coding scheme with examples from both the teacher's and the target student's discourse moves.

Moreover, in order to analyze the linguistic scaffold, we adopted the concordance software AntConc 3.4.3w (Anthony, 2014). AntConc is a useful tool for analyzing a detailed corpus in linguistic research (Lei, 2016). After obtaining the organized discourse coding transcripts from Nvivo, we imported them into AntConc to analyze the frequency of the teacher's language in session transcripts by counting the four categories, such as "How many towers?" "How many cubes?" "How many more?" and "PGBM" (Please Go and Bring Me), that were the major activities involved in the constructivist-oriented teaching experiment for students' learning of multiplicative reasoning and problem solving (Xin et al., 2008).

Table 3. ***Scaffolding and Coding Scheme***

Scaffolds	Teacher	Students
Visual/Graphic Scaffold	"Please generate a model of 5 towers of 9 on the grid ½ sheet."	"Can I use paper to draw a model for this situation?"
Interactive Scaffold	The teacher helps E with the arithmetic and shows her the error she made; now E has 45.	The teacher asked L to help E, and she does. L counts towers for E until L shows 5 with his hand.
Linguistic Scaffold	T: How many cubes do you already know are in a tower? L: 6. T: How many towers in all? E: 5.	L: How many cubes in each tower? E: 5. L: How many towers? E: 6.
Kinesthetic Scaffold	"Use my fingers to keep track of it. And we can use our fingers if it is helpful. Here it is very helpful because you can keep track how many groups you have."	"I counted with my fingers."

In addition, we defined the interactive scaffolds by three characteristics of interaction: teacher-student interaction, student-student interaction and small group interaction (see Table 4).

Table 4. ***Interactive Scaffolds***

Teacher-student interaction	Student-student interaction	Small-group interaction
The teacher helps E with the arithmetic and shows her the error she made; now E has 45.	The teacher asks L to help E, and she does. L counts towers for E until L shows 5 with her hand.	T: How many cubes in all? E: 28. T: OK. What did you get on the calculator? (to L) L: 44 (with calculator).

Limitation

In addition to this special intervention, the school also used two programs for fifth-grade math. First, Math in Focus, a program framed within the Singapore approach, focuses on developing students' conceptual understanding and problem solving (Jaciw et al., 2016). Second, enVisionMath, which is reflective of the Common Core State Standards, and focuses on understanding math concepts through visual instruction and small-group interaction on reasoning and modeling (Charles et al., 2012). This instructional content taught during the school day is a possible variable that might have influenced how students engaged with the PGBM-COMPS intervention in the after-school sessions.

Results

In the first stage of analysis, we report the frequency results for the scaffolds used by the teacher and appropriated by the EL, Eliza. Figure 2 presents the frequency counts of the scaffolds for the focal EL and the intervention teacher. As shown, the highest frequency of scaffolds used by the student and the teacher were kinesthetic scaffolds, while the second-highest was interactive scaffolds. The teacher used finger counting to help students do multiplication to solve the different types of problems, such as unit rate (UR) (e.g., "how many cubes in each tower"); composite units (CU) (e.g., "how many towers"); and 1's (e.g., "how many cubes in all") (Tzur et al., 2010). Students also often used finger counting for multiplication with numbers. Below is an exchange between Eliza (E) and teacher (T).

Excerpt 1 (December 11)

T: Make 8 towers of 7. (Writes it down on the paper. Covers the towers with paper.)

T: Create a model for this situation. Try to solve it without drawing or writing anything down. If you cannot do it, you can write things down. Discuss with each other.

E: 7 plus 7 equals 14 for 2 towers. (Finger counting; counts up to 21. She tries to keep track with her fingers and wanted to be at 7 fingers when she had her answer.)

T: (prompt) Write down the number of cubes you got. Do you want to use my fingers?

E: Yes. (Counts the towers, 28, 35, 42, …)

T: (Explains the number counting method to E using his fingers. E counts and T keeps track of the towers with his fingers – 1 tower of 7.)

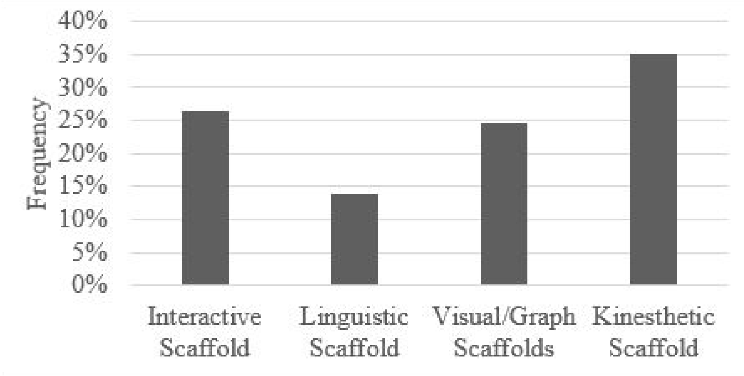

Figure 2. **Frequency of scaffolds across students and the teacher.**

As shown in this example, the teacher prompted Eliza to use both her fingers and the teacher's fingers to solve the problem. In this situation, the interaction between the teacher and the student facilitated understanding. In the interaction, the teacher employed the finger counting method to teach multiplicative reasoning.

Figure 3 shows the different types of interactions that the teacher and students used cumulatively for all sessions. As illustrated, during this teaching event, three areas were used: (a) small-group interaction, which involved interaction among the teacher, Leslie, and Eliza; (b) student-student interaction, which included interactions between students, Leslie and Eliza; and (c) teacher-student interaction between teacher and one student (Leslie or Eliza).

Findings showed that the teacher used more small-group interactions, whereas the students had more interactions during group work with both their classmate and the teacher. For example, the following excerpt is from a transcript between the teacher (T) and students Eliza (E) and Leslie (L).

Excerpt 2 (February 17)

T: Question number one.
L: How many cubes in each tower?
E: 5.
L: How many towers?
E: Six.
T: Six what?
E: Six towers.
L: How many cubes ... in each?
E: 5.
L: How many in all?
T: How many what?
L: How many towers in all?
E: Six.
T: I think the question you're looking for is how many cubes in all Can you ask it?
L: How many cubes in all?
E: 30.

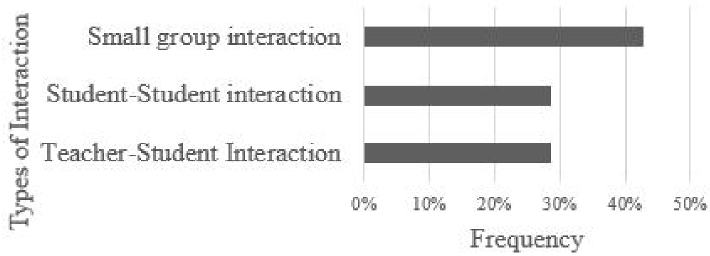

Figure 3. **Frequency of scaffolding used by interaction types.**

The above interaction shows that the EL, Eliza, answered the native English speaker using different types of questions (Unit Rate [UR] and Composite Unit [CU]) in an interactive way to help each other understand the three basic elements (i.e., "UR, # of Units, and Product," Xin, 2012) in elementary multiplicative problem solving. The teacher was involved in the student-student interaction to ensure their linguistic usage was accurate and to check their understanding (such as "how many what?").

Using AntConc, the teacher's language in the session transcripts was found to contain the phrase "how many?" 111 times; "how many towers?" was used 18 times; and "how many cubes?" was used 37 times. Another keyword that the teacher frequently used was "PGBM," or "Please Go and Bring Me," which referred the main student task of the turn-taking platform game PGBM used in the study (Xin et al., 2008). The authors created this game, using a simple language to name it and to make it linguistically accessible for dually classified English learners (e.g., the teacher used statements such as "PGBM a tower of 11," "PGBM 6 cubes" in his instructions). Figure 4 shows the frequency of the language used by the teacher. As illustrated, the teacher used the term "PGBM" most often.

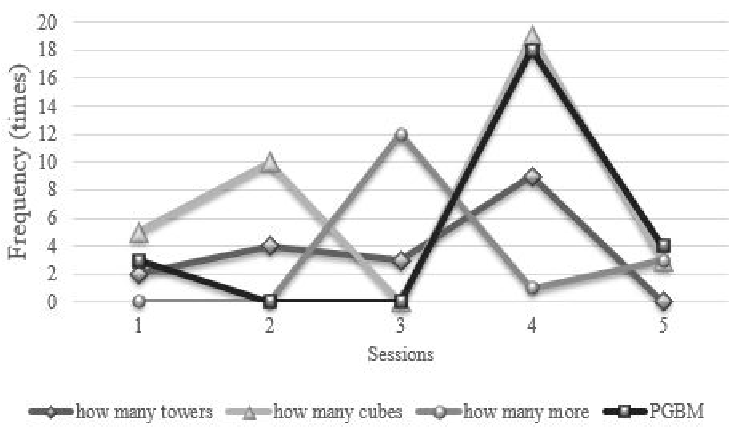

Figure 4. **Frequency of language used by the teacher.**

In the last stage of analysis, the frequency of scaffold usage by the teacher across all seven sessions was analyzed. As shown in Figure 5, during the first session, the most common scaffold used was kinesthetic, but its use was gradually reduced in the following sessions. This change indicates that in the beginning stage, the teacher used more concrete and/or physical scaffolds to support students' construction of a concept. For example, he often used finger counting to help students keep track of the two-unit types involved (1s and composite units larger than 1) as a way to answer questions about the unit rate (UR) (e.g., "How many cubes in each tower?"), number of units (e.g., "How many towers?"), and the product or total number of items (e.g., "How many cubes in all?") (Tzur et al., 2010; Xin, 2012). Students in these sessions often used finger counting to keep track of multiplicative operations on numbers.

But after four sessions, linguistic scaffolds were used more frequently. These shifts reflect that as students acquired more knowledge, the teacher shifted to more abstract approaches.

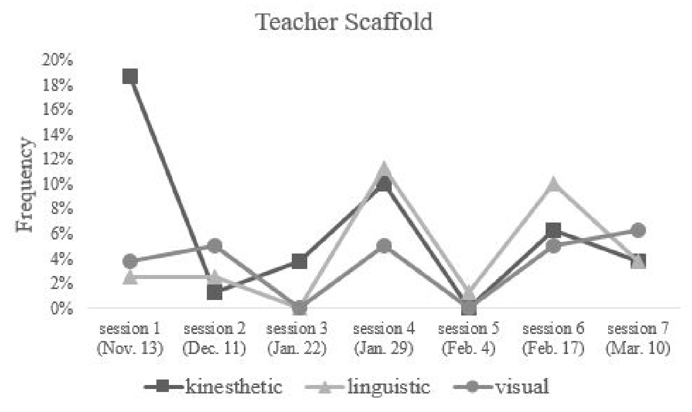

***Figure 5*. Frequency of scaffolds used by the teacher across sessions.**

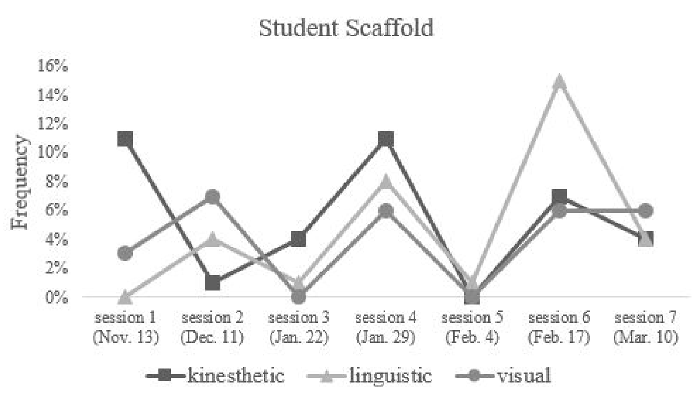

***Figure 6.* Frequency of scaffolds used by the student across sessions.**

Figure 6 shows the frequency of the scaffolds used by students across all seven sessions. Comparison with Figure 5 shows that in the first five sessions, the frequency of linguistic scaffolds students appropriated is lower than that of the teacher. An explanation for the difference may be that students' construction and appropriation of linguistic scaffolds were delayed in relation to the teacher's instruction, as the teacher's instructional levels switched from concrete to abstract. However, by the sixth session, students were able to catch up with the abstract level. Below is an exchange among Eliza (E), Leslie (L), and the teacher (T).

Excerpt 3 (December 11)

T: (Sets out some cubes on the table.) How about we all do 7 towers of 4?

(Students and teacher work separately; Eliza made 3 towers of 4, the teacher put them together.)

T: (Writes on a piece of paper that covers those towers "7 towers of 4," and gives a piece of paper to L and E.)

T: Make a model of 7 towers of 4.

T: How many cubes do we have in all?

E & *L*: (Count on their fingers, then respond.) 28.

T to *E*: How did you get that?

E: (Demonstrates double-counting on her fingers.) 4, 8, 12, until 28.

L: Each tower has 4.

As shown in this example, during session One, the teacher used a linguistic scaffold ("How many cubes ...?") while covering the towers and let students try to figure out the answer without any concrete objects available. However, at this stage, the students did not seem to have sufficient abstract ability to solve the problem, as indicated by their use of fingers (kinesthetic scaffold) to count numbers. Accordingly, the teacher followed by using kinesthetic scaffolds in the first four sessions to facilitate students' gradual transfer to a more abstract level of thinking when solving multiplicative problems. This also met the requirements of linguistic scaffolds (abstract level).

Discussion

The teacher in this study used interactive, linguistic, visual/graphic, and kinesthetic scaffolds in multiplicative reasoning to scaffold instruction for the dually classified EL, Eliza. Among these, the kinesthetic scaffold was the most frequently used. Specifically, the teacher used finger counting to show the student how to solve composite units (CU) and unit rates (UR). Interactive scaffolding, divided into three types: student-student, teacher-student, and small-group interaction, was used with the second-highest frequency.

The results show that small-group interaction was the most effective and useful interaction among the students and the teacher. That is, Eliza, the dually classified EL, performed best in small-group interactions where she demonstrated a greater willingness and capacity to think and answer multiplicative problems.

When the teacher taught multiplicative reasoning to Eliza, he frequently used simple phrases such as "how many?" and "PGBM." It seems that his repeated use of these simple phrases, and having students repeatedly use simple phrases such as "how many?," served as a linguistic scaffolding to help the student understand the three elements (unit rate, # of units, and product; Xin, 2012) in multiplicative reasoning of equal-group problem solving. In addition, "PGBM" characterizes the platform game used, which also benefits ELs to get directions promptly and attend to the target content of multiplicative reasoning.

As illustrated, the four scaffolds in classroom discourse that the teacher frequently used with students influenced the multiplicative reasoning of the EL with learning disabilities and improved mathematical problem-solving achievement (Xin et al., 2017). Kinesthetic scaffolding is the most direct approach to helping dually classified ELs solve multiplicative problems, as evidenced by the density and frequency of use by the teacher and the dually classified EL, Eliza. However, in order to better serve dually classified ELs, especially in the classroom environment, teachers could focus on better linguistic scaffolding use within small-group interactions.

The findings from this study demonstrate three new areas of significance for the fields of special education, English learning, and bilingual education. First, kinesthetic scaffolds are best paired with multiplicative reasoning – math content that is computational but also involves conceptual reasoning. Second, the use of scaffolds ensures that dually classified ELs receive input in a way they can understand, thereby allowing differentiated ways for ELs to express their math comprehension. Third, the scaffolds used by the instructor within the PGBM-COMPS math intervention are well suited for the content area of math. Each of these findings will be discussed in more detail below.

Kinesthetic + Multiplicative Reasoning Equals Greater Student Comprehension and Expression

Visual scaffolds are the most commonly used scaffold with ELs (Walqui, 2010; Walqui & vanLier, 2010). This type of scaffold is easily accessible to teachers; thus, any related content or concept can be matched with a well-selected picture or photo. The use of visual scaffolds is often included in Els' Individual Learning Plans (similar to an IEP within special education), and as a result, teachers use this type of scaffold as a means of adhering to the EL accommodations. However, although a visual scaffold may provide some access for dually classified ELs, it cannot stand alone.

In this study, multiple scaffolds, particularly kinesthetic ones, were used simultaneously or consecutively in combination with intentionally composed language. This crafted approach creates intra-scaffold support, whereby no one scaffold stands on its own. All scaffolds are related to the content, the related math skills, with an aim of comprehension and thereby, student expression or comprehension. As illustrated, Eliza, the dually classified EL, adopted the teacher's scaffolds and used them not just to reproduce what the teacher stated, but to show, act out, state, and perform multiplicative reasoning.

Comprehensible Input Paired With Opportunities for Comprehensible Output

When ELs are taught in a language over which they do not have full mastery, it is important to take into consideration how input (instruction) and output (student production) are understood and expressed. Often, the input that dually classified ELs receive is only partially understood due to their level of language proficiency; as a result, their production of that limited comprehension is a reflection of the original incomplete input (instruction). By expanding the ways in which dually classified ELs experience scaffolded instruction along with student-to-student interaction, their comprehension improves as does the outcome in terms of math comprehension and related academic language. Put simply, math and language are experienced and expressed concurrently when scaffolds are carefully conceptualized and crafted.

Kinesthetic + Other Scaffolds Are Well Suited for Mathematics Content

Math content and related standards lend themselves to the use of a particular type of scaffold; namely, kinesthetic. Because manipulatives or realia were used by the instructor and students and Eliza regularly used finger counting, as expected, our findings showed the impact on Eliza's comprehension and her appropriation of math content and its related language features.

Conclusion and Scholarly Significance

This study provides support for the position that if teachers intersect kinesthetic, linguistic, and visual scaffolds, they can help dually classified ELs to learn relevant content area knowledge while also learning English as it is used within, in this case, mathematical discourse. This way, scaffolding instruction can make English learning and content learning happen concurrently, leading to a model of enrichment instruction vs. remedial instruction. In addition, scaffolding fosters teachers' attention to and awareness of their teaching practices, thereby benefiting dually classified ELs in mathematics.

Few studies have been conducted on instructional interventions for dually classified ELs in the fields of the EL and special education, leading to uncertainty about which interventions concurrently incorporate the content, language and specific strategies needed by these students. As both fields have a distinct repertoire of instructional provisions proven to be effective, collaboration is essential for truly understanding the role of these provisions for each field. As illustrated in this study, the use of scaffolds, long used in special education and newly incorporated into EL education, is a shared pivot point from where decisions can be developed for better instruction for ELs with LD.

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