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Examination of Cognitive Processes in Effective Algebra Problem-Solving Interventions for Secondary Students with Learning Disabilities

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Algebra problem solving is one of the most difficult areas in the mathematics curriculum for secondary students with learning disabilities (LD) due to the higher-order reasoning demands and strategic thinking required. The purpose of this review is to examine how effective algebra problem-solving interventions conceptualize the cognitive processes of problem solving, and to examine the types of instructional supports or strategies embedded in each problem-solving phase to facilitate cognitive processes. In 11 effective algebra interventions, we identified four conceptualizations of the cognitive processes involved in problem solving: (a) sequential concrete–semi-concrete–abstract, (b) sequential virtual-abstract, (c) integrated concrete–semi-concrete–abstract, and (d) abstract only. We also found that each intervention incorporated several instructional strategies (i.e., scaffolds) to support students through the cognitive process of problem solving. Educational implications, future directions, and limitations are discussed.

Keywords: **algebra, problem solving, cognitive process, LD**

Algebra is considered the gatekeeper to higher-level mathematics and to opportunities in educational achievement and postsecondary occupations (Fennell, 2008; Foegen, 2008; National Mathematics Advisory Panel [NMAP], 2008). In addition, passing Algebra I is a requirement for high school graduation in a growing number of school districts across the United States (Witzel, 2005). According to the American Diploma Project Network, 22 states required Algebra I and one state required Algebra II for high school graduation in 2009. By 2018, the number of states requiring Algebra I and Algebra II for graduation is anticipated to increase to 29 and 19, respectively. In addition, many studies have emphasized the importance of teaching and learning mathematics skills, as they are a key requirement for a diverse array of occupations, especially for many high-wage jobs that do not require a two- or four-year degree (Hwang & Riccomini, 2016; Cavanagh, 2007; Gonzalez & Kuenzi, 2012). In effect, because understanding elementary algebraic concepts is necessary for solving various advanced mathematical problems, helping students achieve algebra

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competence increases the probability that they will graduate from high school and have access to a range of post-secondary education and employment opportunities.

Given the importance of algebra, it is critical to identify effective practices for improving the performance of all students, especially those who struggle in mathematics. The need for special attention to students with learning disabilities (LD) has been evidenced by the large algebra achievement gap. In the 2013 National Assessment of Educational Progress (NAEP), there was a 46-point discrepancy between average algebra subtest scores for students with and without disabilities in eighth grade, and a 40-point difference between average algebra scores in 12th grade. Research suggests that characteristic deficits in working memory, long-term memory, and organizational skills contribute to the relatively weak algebra performance of students with LD (e.g., Geary, Hoard, Nugent, & Bailey, 2012; Scanlon, 2013). These deficits affect the basic mathematics difficulties of students with LD beginning in elementary school and continue through middle and high school with increasingly negative effects in advanced domains (Cawley & Miller, 1989; Impecoven-Lind & Foegen, 2010; Miller & Mercer, 1997).

Mathematical Problem Solving in Algebra

Problem solving is a complex skill that requires students to understand and integrate information from the problem, create a mental representation of the problem, and choose an appropriate solution path (Hudson & Miller, 2006; Montague, Warger, & Morgan, 2000). Problem solving in algebra can be especially challenging. Higher-order reasoning demands and strategic thinking required make problem solving tasks some of the most difficult tasks in the mathematics curriculum, especially for students with identified LD (Lerner, 2000). Despite its importance, algebraic problem solving is an ambiguous construct, and various definitions exist. In some instances, problem solving refers to a single behavior and is evaluated simply by the correctness of problem solutions. In others, problem solving refers to the accurate completion of the many steps involved in reaching a correct answer. In addition, determinations about problem-solving ability are sometimes made based on students' performance on one type of problem. A common example is the practice of using problem solving as a generic label for word-problem solving.

Cognitive process of problem solving. Although a uniform definition of problem solving has not been established, there is general consensus that complex problem solving requires an integrated set of skills, while simple problem solving requires a monolithic ability such as ordering or comparing numbers. Mayer (1987, 1989, 1992, 1998) defined problem solving as cognitive processing directed at achieving a goal when no solution method is obvious to the problem solver and defined four key characteristics of problem solving. First, the work of problem solving is *cognitive* and is inferred indirectly from the problem solver's behavior. Second, problem solving is *a process* involving representing and manipulating knowledge in the problem solver's cognitive system. Third, problem solving is *goal directed* because a problem solver aims to achieve the goal of solving the problem. Fourth, problem solving is *personal* because problem solvers' individual experiences and skills determine their perceptions of the difficulty or complexity of a problem.

In addition, Mayer (1987) proposed a theoretical model to describe the underlying cognitive process (and sub-processes) of problem solving. In this model, a problem solver begins with problem representation and progresses to the problem solution. During the problem representation phase, students interpret problem situations or statements and transform the problem with various types of representations (e.g., concrete objects, symbolic images or sketches, mental representations, or mathematical notations). Next, students integrate their representations so that they have a coherent understanding of the problem. In the problem solution phase, students devise and execute a solution plan to achieve a goal (Mayer, 1987, 1998). Based on an understanding of a problem that the student developed in the representation phase, the student determines the appropriate operation to apply and carries out the solution plan (e.g., sequence of a solution procedure, number and order of steps required for the procedure, and computation). The solution phase reflects students' knowledge of the various possible ways to solve a problem and reveals their chosen method. For example, when students solve an algebra problem involving fractions to compute the unknown value x (e.g., $x - {}^{1}/_{3} = 4\ {}^{2}/_{3}$), they may want to rename a mixed number ($4\ {}^{2}/_{3}$) as an improper fraction (${}^{14}/_{3}$) first and do the operations next ($x = {}^{1}/_{3} + {}^{14}/_{3} = {}^{15}/_{3} = 5$), or they may want to do whole number operations and fraction operations separately ($x = {}^{1}/_{3} + 4\ {}^{2}/_{3} = (0+4) + ({}^{1}/_{3} + {}^{2}/_{3}) = 4\ {}^{3}/_{3} = 5$) to achieve an answer.

Structure of the Current Study

Recent work (Hughes, Witzel, Riccomini, Karen, & Kanyongo, 2014; Watt, Watkins, & Abbitt, 2014) has provided overviews of algebra interventions for students with disabilities. In both reviews, researchers categorized interventions and compared intervention effectiveness by intervention type. All of the reviewed interventions were effective, but comparisons indicated that cognitive model-based interventions were most effective. Rather than conduct another comparison of intervention effectiveness, the current study took a different approach and examined the structure of effective interventions in light of cognitive processes. We based our analysis on Mayer's (1987) cognitive model, which considers algebra problem-solving interventions to be cognitive processes; and modeled each intervention as a combination of subsystems that are composed of a main system of cognitive processes and incorporated instructional support facilitating these cognitive processes. The structure of intervention was conceptualized as $T(x) = y$, where x denotes inputs (i.e., algebra problems) and y denotes outputs (i.e., achievement). Sequential cognitive processes, problem representations, and problem solutions (Mayer, 1987) were considered connected subsystems and represented as $T_S(T_R(x)) = y$. We concentrated on examining different types of cognitive systems, T, consisting of effective algebra interventions along with instructional components that are embedded into intervention systems.

The present study is concerned with two aspects of algebra problem-solving interventions: (a) cognitive processes and (b) instructional supports. Based on the knowledge that the cognitive problem-solving process consists of two primary phases, problem representation and problem solution, we analyzed how effective interventions specify each of phase and what types of instructional supports or strategies were embedded within each phase. The purposes of this review were: (a) to examine how effective algebra problem solving interventions conceptualize and define the problem representation and problem solution phases within the cognitive process of problem solving; (b) to categorize interventions by the characteristics of the cognitive problem solving process students are taught to use; and (c) to examine the types of instructional support used in each problem solving phase to facilitate cognitive processes. The following research questions guided the review:

1. What models of cognitive processing have been used to conceptualize effective algebra problem-solving interventions for secondary students with LD?
2. Within each category of cognitive process of a problem-solving intervention, what types of instructional supports have been embedded in order to facilitate the cognitive problem-solving process?

Method

Search Criteria and Procedure

The current study examined and integrated recent peer-reviewed experimental studies that have addressed the significant effectiveness of algebra problem solving interventions for middle school and high school students with LD. Three criteria were used to identify studies for inclusion. First, intervention studies must have included algebra content, regardless of problem type (e.g., word problems or computation). Although there is a common conception that problem solving is limited to word-problem solving, we also included computation problem solving interventions because, according to Mayer (1987), two phases of the cognitive process should always occur when solving any type of mathematical problem, not only word problems. Although pre-algebraic ideas and basic skills are broadly taught during the elementary period (e.g., whole number operations, conceptual knowledge of rational numbers), we did not include studies testing interventions of pre-algebraic skills in the current review because the use of unknown variables is not fully considered or introduced in elementary mathematics (Common Core State Standard Initiative [CCSSI], 2012; Miller & Mercer, 1993).

We included studies that tested intervention effects on problem solving or application of "algebraic knowledge," so we excluded intervention studies designed exclusively to increase procedural skills and those focused on academic behavior (e.g., finding keywords and mnemonic-only and peer-tutoring strategies; e.g., Allsopp, 1997). In addition, we only included interventions that reported positive effects on algebra problem solving. In single-case design studies, effects were examined with systematic visual inspection of level, trend, and variability in data within and across study conditions (Horner et al., 2005) and/or statistical calculations (e.g., percentage of nonoverlapping data and Tau-U) when provided. We defined a positive effect as an increase in the means from the baseline to the intervention conditions. In group design, positive effects were defined as a significant mean difference or change between pre- and posttest conditions or control and intervention group performance.

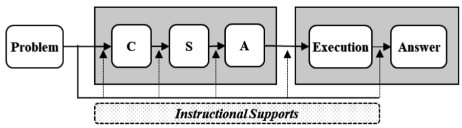
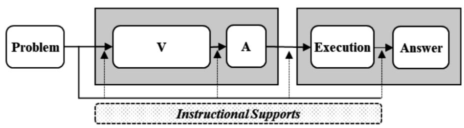
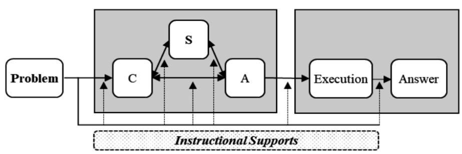
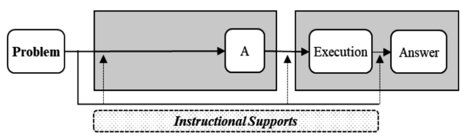
Second, studies published after 1990 were included to capture research reflecting the standards outlined in the National Council of Teachers of Mathematics Principles and Standards publication (NCTM, 2008, 2014), as trends and educational philosophies have changed over time. Third, participants were limited to U.S. middle and high school students in Grades 6 to 12 identified as having LD. This study used the Individuals with Disabilities Education Improvement Act (IDEA, 2004) definition of LD, in which LD is described as a psychological processing disorder. Participants in included studies were identified as having LD using one of the three methods depending on the method used in their home state (ability-achievement discrepancy model, response-to-intervention model, and patterns of strengths and weaknesses). We excluded studies with participants at the college level because our primary interest is in algebra intervention for school-aged students.

Using these three criteria, a systematic literature search was conducted to identify potential articles for inclusion. First, three electronic databases, ERIC, ProQuest, and PsychINFO, were used with a combination of the following descriptors in abstracts: learning disab*, learning struggles, learning difficult*, math disab*, math*, algebra*, middle school, high school, secondary school, problem-solving. Additionally, well-published researchers in the area of algebra, including Maccini, Montague, and Strickland, were used as search descriptors. Second, an ancestral search as well as an examination of the reference lists of relevant studies (e.g., Hughes et al., 2014) were conducted. Third, a hand search of the major journals in the areas of special education, learning disabilities, mathematics education was conducted that included the following journals: *Education & Treatment of Children, Exceptional Children, Learning Disabilities Quarterly, Learning Disabilities Research & Practice, Journal of Learning Disabilities, Journal of Special Education, Remedial and Special Education, Learning Disabilities Contemporary Journal,* and *Learning Disabilities: A Multidisciplinary Journal.* As a result, a total of 10 studies met the inclusion criteria for this review. One study (Ives, 2007) was composed of two sub-studies (Studies 1 and 2) to provide systematic replication, where Study 2 investigated extended skills and content with different participants. Both of Ives's (2007) sub-studies were considered independent studies, so 11 potential studies were identified. After we examined all 11 interventions and found that they were effective in increasing algebra problem solving performance, they were all included in the review.

Results

Among the 11 studies, more than 80% of students were in Grades 6 to 8, indicating that studies on algebra intervention for students with LD have been more heavily focused on middle school than high school. Two types of research design emerged: single-case design (Huchinson, 1993; Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Scheuermann, Deshler, & Schumaker, 2007; Strickland & Maccini, 2012) and group design (Huchinson, 1993; Witzel, Mercer & Miller, 2003; Ives, 2007; Witzel, 2005). All the interventions in identified studies were effective in that they showed a significant difference between baseline (pre-test) and intervention (post-test) conditions and/or a difference between control and intervention groups that favored intervention. We did not report and combine effect sizes by intervention type because there have been arguments against finding a common effect size metric between the two research methods and among the different types of dependent measures the studies employed (e.g., accuracy percentage and number of problems correct). More importantly, the main purpose of this study was to examine the features of cognitive processes and instructional supports that composed effective algebra interventions, rather than comparing effectiveness across intervention types. In order to investigate and answer the research questions, the 11 identified studies were examined in terms of (a) cognitive process of algebra problem-solving intervention and (b) instructional supports embedded in each intervention type.

Table 1. ***Cognitive Process of Algebra Intervention***

Cognitive Process Type	Study	Two Phase of Cognitive Process: Problem Representation	Two Phase of Cognitive Process: Problem Solution
Sequential C-S-A	Hutchinson, 1993; Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Witzel et al., 2003; Scheuermann et al., 2007; Witzel, 2005		
Sequential V-A	Satsangi & Bouck, 2015; Shin & Bryant, 2017		
Integrated C-S-A	Strickland & Maccini (2012)		
A-only	Ives (2007)		

Note. C-S-A=concrete–semi-concrete–abstract intervention; V-A= virtual-abstract

Cognitive Process of Algebra Interventions

The seven intervention types that emerged in the 11 included studies were classified by the features of their cognitive process, particularly how they specified each phase of the problem-solving process (i.e., problem representation and problem solution) in order to structure a connected system. As a result, we found four systems of cognitive processing: (a) sequential concrete–semi-concrete–abstract intervention (C-S-A; Hutchinson, 1993; Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Scheuermann et al., 2007; Witzel et al., 2003; Witzel, 2005); (b) sequential virtual-abstract intervention (V-A; Satsangi & Bouck, 2015; Shin & Bryant, 2017),;(c) integrated concrete–semi-concrete–abstract intervention (C-S-A-I; Strickland & Maccini, 2012); and (d) abstract-only intervention (A-only; Ives, 2007).

As shown in the Table 1, the problem representation phase served as a broad category for representing problems that allow for any type of representation. All interventions except abstract-only interventions broke down the problem representation phase into several sub-phases. In C-S-A and C-S-A-I interventions, problem representations were divided into three stages (concrete, semi-concrete, and abstract) either sequentially or simultaneously; while in A-only interventions, the cognitive process was fixed to the abstract stage with symbolic manipulations only. Meanwhile, in sequential V-A interventions, concrete and semi-concrete representation stages (C and S, respectively) were replaced with a virtual representation stage (V), where students were provided a virtual space to manipulate and represent problems on a computer screen. Each cognitive processing intervention category is described in the next section along with embedded instructional supports.

Instructional Support

In addition to the intervention categories characterized by a cognitive process, we examined the instructional supports or strategies integrated within each intervention that were used to support the cognitive problem-solving process. Results indicate that all interventions used an explicit instruction lesson format. In all studies, one or more of the six components of explicit instruction (advance organizer, modeling, guided practice, independent practice, post-test, and feedback/rewards) were integrated through the entire cognitive process. In some studies (e.g., Hutchinson, 1993; Maccini & Hughes, 2000; Ives, 2007), lesson plans were scripted to facilitate instructional procedures and fidelity.

Sequential C-S-A. The sequential C-S-A cognitive process intervention category specifies that the problem representation phase has sequential stages: concrete (C) and semi-concrete (S). Interventions in the sequential C-S-A category involved a three-stage, graduated instructional sequence beginning with manipulation of concrete objects (C), followed by using drawings or pictorial representation (S), and ending with students using abstract mathematic symbols or notation (A). After students represented problems following the three sequential representation stages, they chose and applied operators to solve problems in the problem-solution phase. Six studies (Hutchinson, 1993; Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Witzel et al., 2003; Witzel, 2005; Scheuermann et al., 2007) used sequential C-S-A interventions, but these interventions included a variety of different types of instructional supports. Two of the six studies (Witzel et al., 2003; Witzel, 2005) used C-S-A exclusively, while the remaining four studies (Hutchinson, 1993; Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Scheuermann et al., 2007) embedded instructional supports and strategies within C-S-A. Witzel et al. (2003) and Witzel (2005) tested the effects of C-S-A interventions on students' algebra performance with no embedded instructional strategies or supports. They particularly focused on algebraic transformation of equations with a single variable. The interventions in both studies consisted of 19 lessons that included five transformation skills to solve equations (e.g., reducing expressions and solving inverse operations).

While two studies examined effectiveness of using only C-S-A, Hutchinson (1993) combined meta-cognitive strategies, such as structured worksheets, self-questioning prompt cards, and lesson scripts, with C-S-A sequence interventions in order to help students shift from the problem representation stage to the problem solution stage. In addition, three studies (Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Scheuermann et al., 2007) employed C-S-A interventions that also incorporated other problem-solving strategies. The C-S-A cognitive process was inherent in the problem-solving process and provided the framework for students to develop solution plans for algebra problems, while strategies and supports were embedded to make the process more efficient. These added strategies supported and catalyzed the cognitive process of the incremental steps in the C-S-A sequence in order to move students into the problem solution phase and also support them to become self-directed learners. Two of these three studies (Maccini & Hughes, 2000; Maccini & Ruhl, 2000) combined C-S-A with the STAR strategy (Search the word problem, Translate the words into an equation in picture form, Answer the problem, and Review the solution; Maccini, 1998), which is a mnemonic device and a type of meta-cognitive strategy that includes a self-monitoring component and helps students remember the sequence of problem solving steps. Each step in the STAR strategy is repeated for every phase of C-S-A. There was a small variation between the two studies in how each applied STAR to the stage of C-S-A. Maccini and Hughes (2000) applied only the first two steps of STAR (S and T) in the S stage, but used all four steps (S, T, A, and R) in the C and A stages; while Maccini and Ruhl (2000) applied S and T in both the C and S stages and every step of STAR was in the A stage.

Similar to the application of C-S-A with the STAR strategy, Scheuermann et al. (2007) integrated the explicit inquiry routine strategy and C-S-A, where three steps of a scaffolded inquiry process (tell me how, tell your neighbor how, tell yourself how) were woven into the intervention to facilitate the C-S-A process. Intervention content was broken down into instructional bites, and for each instructional bite, the students and teacher worked through the steps of scaffolded inquiry within each phase of C-S-A. This process was designed to facilitate the transition of student's cognition through the C, S, and A phases of representation. Instruction then proceeded to the next instructional bite and students were expected to independently demonstrate mastery within C-S-A. This three-step scaffolded inquiry process is similar to STAR in that both strategies function as scaffolds to promote students' cognitive problem-solving process; however, the scaffolded inquiry routine incorporates a supportive scaffold into the explorative nature of inquiry (Coyne, Kameenui, & Carnine, 2007; Goos, 2004; Rosenshine, & Meister, 1992).

In four studies (Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Witzel et al., 2003; Witzel, 2005), we observed a difference in the method used in intervention instruction for representing coefficients and variables during the problem representation phase. In two studies, coefficients were represented with the number of variables during the first two stages (C and S; e.g., $5x$ is represented by "five x"; Maccini & Hughes, 2000; Maccini & Ruhl, 2000), while in two other studies the coefficient and variable were represented as independent units (e.g., $5x$ is represented by "5" and "x"; Witzel et al., 2003; Witzel, 2005). Authors of another two studies insisted that representing coefficients by the number of variables might induce confusion in algebra learning because students learn to differentiate coefficients from exponents when they learn to solve more complex equations. In response to this issue, two studies (Witzel et al., 2003; Witzel, 2005) suggested representing coefficients and variables independently by considering each a separate unit, so that every component of linear functions are expressed and arranged. For example, when given "$5x = 15$," the coefficient marker for 5 and blocks for the variable x are used separately, instead of arranging five blocks to represent $5x$. When representing the other components, such as the equal sign or the constant, 15, four studies (Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Witzel et al., 2003; Witzel, 2005) were done in the same way.

Sequential V-A. Two studies (Satsangi & Bouck, 2015; Shin & Bryant, 2017) were categorized as using the sequential V-A cognitive process, where concrete manipulatives were replaced with virtual manipulatives. This practice reflects the increased use of computer-based technology in education. V-A interventions used the same framework for the cognitive problem solving process as sequential C-S-A interventions, but the platform for problem representation was transferred to a virtual space. The virtual space was provided via a software program or an internet website. When given an algebra problem, students manipulated the shapes on the screen that were needed to solve the problem. For example, they drew shapes, increased or decreased the size and color of blocks, and moved blocks around to rearrange them. Prior to the intervention in both studies, teachers provided instructional lessons about basic concepts of algebraic topics (e.g., area and perimeter), reviewed essential mathematics vocabulary, and modeled how to solve problems using virtual manipulatives. During the intervention, students' work was guided by the teachers' instructions in a step-by-step manner.

In both studies, several instructional components were used in conjunction with sequential V-A. Satsangi and Bouck (2015) concentrated on the sole effect of virtual manipulatives, but used teacher modeling, guided practice, feedback, and independent practice to support student learning in a virtual environment. Shin and Bryant's (2017) intervention also focused on the use of virtual manipulatives to transition students' reasoning and thinking to abstract mathematics, but they examined the combined effect of multiple intervention components, including cognitive and metacognitive strategies, explicit and systematic instruction, and virtual manipulatives. The effectiveness of their intervention could have been derived from the interplay between the various components integrated into the intervention.

Integrated C-S-A (C-S-A-I). An integrated version of C-S-A, C-S-A-I, modifies the C-S-A cognitive process by implementing the three stages simultaneously (rather than in sequence). The integration and simultaneous presentation of the stages is the biggest difference between C-S-A-I intervention and the eight interventions that involved sequential cognitive processes (C-S-A and V-A). In C-S-A-I interventions, the three stages of representation were not introduced linearly over time, but presented simultaneously, which promoted interaction and successful transfer among the C, S, and A stages. The intervention used in Strickland and Maccini's (2012) study employed the C-S-A-I cognitive process and integrated a graphic organizer (in this case, a box plot) along with explicit instructional components as an instructional support. The graphic organizer helped students visually organize and remember the steps of the problem-solving process, and supported students in learning to alternate between representation phases.

A-Only. Ives's (2007) two studies tested interventions where only an A phase representations were used. Unlike C-S-A, V-A, and C-S-A-I, the problem representation phase was not specified, so students did not have opportunities to use scaffolded C or S representations of the problem. Instead, to find the value of an unknown variable, students were only provided a graphic organizer to help organize the symbolic notation in complex algebraic manipulations. Each cell in the graphic organizer functioned as a flow chart to help students solve equations by providing a sequence of steps. Although students' cognition was limited to the A stage while solving problems, the graphic organizer provided graphical scaffolding to reduce cognitive load by dividing complex problem solving procedures and translating verbal elements into graphic ones (e.g., box plot, arrows, and lines).

Discussion

Mathematical problem solving requires an integrated set of mathematical knowledge and skills, and various factors (e.g., literacy, computation skills, and cognitive strategy) contribute to students' poor achievement in problem solving (Hwang & Riccomini, 2016; Fuchs & Fuchs, 2002; Vilenius-Tuohimaa, Aunola, & Nurmi, 2008; Xin, Jitendra, & Deatline-Buchman, 2005). Students' difficulties often stem from the fact that they do not know *how* to execute or move through the sequential cognitive phases of problem solving and do not know *what* to do within each phase. In response to the different types of barriers that students' face during problem solving, problem-solving interventions have employed various methods and strategies. Some interventions specify each phase of the cognitive problem-solving process, where each stage scaffolds students' ability to translate a problem to the next stage (e.g., in C-S-A, the concrete stage prepares students for the semi-concrete stage, which in turn prepares students for the abstract stage). In addition to refining a cognitive process based on students' needs, some interventions set up instructional scaffolds to help students proceed through the process. For example, some interventions include cognitive and/or meta-cognitive strategies to guide students through the problem-solving process (e.g., mnemonics, self-question, and scripted notes; Montague, 2007), while others include organizational strategies like cue cards and graphic organizers (e.g., Ives, 2007) to facilitate the cognitive processes required for problem solving. Although the components of interventions and their emphases may vary, we characterized the included algebra problem-solving interventions by the features of the cognitive process as it progresses from a problem to a final answer, and examined the additional instructional supports or strategies used to scaffold students' ability to move through the problem-solving process.

Four Meaningful Observations

The analysis of the features and constructs of effective algebra problem-solving interventions led us to four observations regarding (a) cognitive process categories that have been effectively employed in algebra problem-solving interventions for secondary students with LD (*research question 1*) and (b) types of instructional supports and scaffolds embedded within interventions to facilitate the cognitive problem-solving process (*research question 2*).

Cognitive process type. First, we identified four cognitive process categories in effective algebra interventions (sequential C-S-A, sequential V-A, C-S-A-I, and A-only). These cognitive processes contain the two main phases (problem representation and problem solution). Janvier (1987) referred to problem representation as converting a problem from words into an internal representation with a combination of something written on paper, something that exists in the form of physical objects, and a carefully constructed idea in one's mind. Based on the representation students create, they move to the problem solution phase, where they apply operators and find a solution (Mayer, 1987, 1989; Reif & Heller, 1982). The whole thought process involved in solving a problem is defined as a cognitive process. The cognitive problem-solving process goes through several consecutive stages in which its dimensions are reduced until it approaches the symbolized, abstract world.

Traditionally, rather than providing gradual steps such as concrete or pictorial manipulation, algebra instruction has merely focused on symbolic manipulations executed through repeated practice while solving abstract equations. Therefore, the cognitive process has been fixed and limited to the abstract stage. Recognizing the increasing achievement gap between students with and without LD and the continued difficulties of students with LD in algebra learning, recent reform in mathematics education has led to the adoption of a functions-based approach to algebra. The functions-based approach focuses on providing multiple representations to promote proficiency with procedures and understanding of algebraic concepts. Reflecting this trend, advocates of sequential C-S-A intervention (Hutchinson, 1993; Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Scheuermann et al., 2007; Witzel et al., 2003; Witzel, 2005) have emphasized adding a problem representation phase prior to applying abstract mathematical operators in the problem solution phase. The representation phase was divided into three gradual stages: concrete, semi-concrete (i.e., representational, pictorial), and abstract. Sequential C-S-A interventions are considered multisensory as their procedure relies on visual, auditory, kinesthetic, and tactile interaction with content.

Second, interventions in the C-S-A and C-S-A-I cognition process categories are similar in that they both teach students to use concrete, semi-concrete, and abstract representations. However, C-S-A and C-S-A-I interventions differ in that C-S-A interventions transition students through the stages in a sequential fashion, while C-S-A-I interventions engage students in all three stages simultaneously. These approaches represent different ways of processing information (sequentially or simultaneously). Sequential and simultaneous processing capacity can be measured using the KABC (Kaufman Assessment Battery for Children). The KABC includes two types of mental processing scales (sequential and simultaneous processing) to measure students' overall intelligence. The sequential processing scale primarily measures short-term memory and consists of subtests that measure problem-solving skills where the emphasis is on following a sequence or order. The simultaneous processing scale examines problem-solving skills that involve several processes at once. Two students could achieve similar intelligence ratings, but have differing strengths and weaknesses on the two scales. In other words, some students may have strength in processing information sequentially, while others may be better at processing information simultaneously. Given their strengths and weaknesses in sequential and simultaneous processing, C-S-A or C-S-A-I may be more effective for any particular student.

Many studies (e.g., Witzel, Riccomini, & Schneider, 2008; Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Flores, Hinton, & Strozier, 2014; Miller & Kaffar, 2011) have supported the use of a gradual sequence of representations (e.g., C-S-A and S-A) for teaching a variety of mathematics content. In addition, an increasing number of studies have suggested that C-S-A-I interventions are a promising approach (e.g., Pashler et al., 2007). Yet, there has been a lack of evidence to show which approach is more effective for teaching algebra to students with LD. Knowing that students with LD show a wide spectrum of learning characteristics and diverse abilities in processing information, it is clear that more work is needed to develop and identify interventions that consider students' cognitive strengths and preferences and the needs of teachers.

Third, the use of virtual manipulatives has been replacing concrete and/or semi-concrete representations. Although a large body of research supports sequential representation interventions (e.g., C-S-A interventions), there is continuing controversy regarding the efficiency and efficacy of using concrete manipulatives. In terms of efficiency, secondary math teachers rarely use concrete objects, in part because they are concerned that objects may not accurately represent a concept (Howard, Perry, & Conroy, 1995). In particular, it can be challenging to represent complex equations with concrete objects. In terms of efficacy, an increasing number of studies have reported a positive impact on the algebra learning of students with LD when virtual manipulatives are used (e.g., Bouck & Flanagan, 2010), and researchers have speculated that virtual manipulatives are particularly suited for students with LD because they decrease cognitive load (Satsangi & Bouck, 2014).

Instructional support. Fourth, included interventions incorporated several different instructional strategies (i.e., scaffolds) to support students' progress through the cognitive problem solving process. All included interventions used two or more components of explicit instruction (advance organizer, modeling, guided practice, independent practice, post-test, and feedback/rewards), and some interventions also included cognitive, meta-cognitive, or other strategies (e.g., graphic organizers, structured worksheets, prompt cards, and mnemonics). All of these strategies were designed to support students as they transitions from phase to phase of the cognitive process. In addition, strategies broke instructional tasks into smaller steps and promoted meta-cognition and self-regulated behavior during problem solving.

Each intervention was comprised of multiple instructional components, which made it challenging to determine exactly which components or combination of components worked to increase student achievement. The effects of included intervention studies may be due to the interplay between components. Therefore, our analysis of disassembling interventions into cognitive processes and supporting instructional strategies contributes to the field because our findings can serve as a basis to examine specific effective components rather than a combination of components, i.e., an intervention. The current review of algebra interventions for students with LD has two educational implications. First, teachers are encouraged to choose and apply interventions that consider students' cognition process tendencies and that include additional instructional strategies as needed. Second, additional research should address the relationship between students' cognitive process system preference (e.g., sequential vs. integrated or C-S-A vs. V-A) and efficacy of student learning. This research would greatly benefit students with LD by shedding light on effective practices for accommodating student's unique cognitive learning characteristics.

Limitations

Three potential limitations of the review were found. First, "algebra" has an ambiguous definition, so only studies that include algebra as a key term were included. Depending on how algebra is defined and viewed (e.g., whether all word-problem solving is considered algebra, as it includes finding unknown variables), future studies can expand on the present review by addressing this gap. Second, due to the purpose of the present review (examination of the cognitive processes used in effective interventions), effect sizes were not calculated within and across intervention types. Third, we only included published studies that reported positive intervention effects and assumed each study was methodologically sound and that results were valid.

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